Completeness:- An LTI system G with a state model (A, B, C, D) is said to be complete if

$$\text{rank}\begin{pmatrix} \begin{bmatrix} A - zI & B \\ C & D \end{bmatrix} \end{pmatrix} \geq n \quad , \forall z \in \mathbb{C}$$

if the condition holds $\forall z \in \overline{\mathbb{D}}$

G is said to be weakly complete.

"essentially combines stabilizability & detectability"

Not complete $\Rightarrow$ not controllable & not observable.

Complementary Subsystems

$\Sigma_{i_1 \cdots i_\mu j_1 \cdots j_\mu}$

State model

$$\left(A, \begin{bmatrix} B_{j_1} & \cdots & B_{j_{\nu-\mu}} \end{bmatrix}, \begin{bmatrix} C_{i_1} \\ \vdots \\ C_{i_\mu} \end{bmatrix}, \begin{bmatrix} D_{i_1 j_1} & \cdots & D_{i_1 j_{\nu-\mu}} \\ \vdots & & \vdots \\ D_{i_\mu j_1} & \cdots & D_{i_\mu j_{\nu-\mu}} \end{bmatrix} \right)$$

$i_k, j_\ell \in \{1, 2 \cdots \nu\}$

$\mu = 1, 2 \cdots \nu - 1$

$\{i_1, \cdots i_\mu, j_1, \cdots j_{\nu-\mu}\} = \{1, 2 \cdots, \nu\}$.

There are $2^\nu - 2$ such subsystem for Σ.