## Detectable

States of the system are internal. All an external observer can see is the output

$$x(k+1) = Ax(k) \qquad x(0) = x_0$$
$$y(k) = Cx(k)$$

$A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{p\times n}$, $y \in \mathbb{R}^p$, $x \in \mathbb{R}^n$

If by observing $y(k)$ over a time interval, we can determine the value of $x_0$. The pair $(C, A)$ is called **Observable**. $\left[\text{rank}\begin{bmatrix} C \\ CA \\ CA^{n-1}\end{bmatrix} = n\right]$

Again the system can be rewritten as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1)\end{bmatrix} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k)\end{bmatrix}$$

$$y(k) = C_1 x_1(k)$$

$(C_1, A_{11})$ is Observable.

Now if $x_1(0) = 0 \Rightarrow x_1(k) = 0 \quad \forall k > 0$

$\Rightarrow y(k) = 0, \quad \forall k > 0$

but $x_2(k+1) = A_{22}x_2(k)$

if $A_{22}$ has all its eigen-values in $\mathbb{D}$, then we can conclude that

$x_2(k) \longrightarrow 0$

∴ we call this system Detectable

**Thm:** The pair $(C, A)$ is detetable iff

$$\text{rank}\begin{bmatrix} A - zI \\ C\end{bmatrix} = n, \quad \forall z \in \bar{\mathbb{D}}$$