Stabilizable

→ lets first look at Controllability

$$x(k+1) = Ax(k) + Bu(k), \quad A \in \mathbb{R}^{n\times n}, \; x(k)\in\mathbb{R}^n, \; u(k)\in\mathbb{R}^m, \; B\in\mathbb{R}^{n\times m}$$

we say the pair (A, B) is controllable

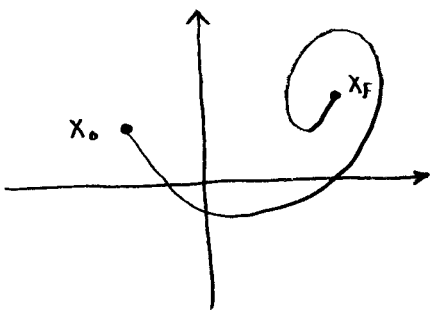
iff $\text{rank}\,[B \;\; AB \;\; \cdots \;\; A^{n-1}B] = n$

→ physically if (A, B) is controllable

Given x_0, x_f & $N \in \mathbb{Z}^+$, & $x(0) = x_0$

$\exists$ a seq. $u(0) \ldots u(N-1)$

s.t. $x(N) = x_f$



→ Now any system P can be written as (by a coordinate transformation)

$$P: \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(k)$$

where (A_{11}, B_1) is controllable

Now we don't have any control over the states $x_2(k)$.

So we call the system stabilizable if all eigenvalues of A_{22} lie in the open unit disk D.

Thm: The pair (A, B) is stabilizable iff

$$\text{rank}\,[A - zI \;\; B] = n, \quad \forall\, z \in \bar{D}$$